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Recovery of full $N = 1$ supersymmetry in non(anti-)commutative superspace

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Abstract: We investigate SUSY of Wess-Zumino models in non(anti-)commutative euclidean superspaces. Non(anti-)commutative deformations break 1/2 SUSY, then non(anti-)commutative Wess-Zumino models do not have full SUSY in general. However, we can recover full SUSY at specific coupling constants satisfying some relations. We give a general way to construct full SUSY non(anti-)commutative Wess-Zumino models. For a some example, we investigate quantum corrections and β-functions behavior.

Keywords: Renormalization Group, Supersymmetry Breaking, Superspaces, Non-Commutative Geometry.

Contents

1. Introduction

Non(anti-)commutative superspaces have attracted much interest [1]–[60]. One of significant features of non(anti-)commutative field theories is 1/2 SUSY. Non(anti-)commutative theories do not have full SUSY, but one half of SUSY is preserved in many cases. Non(anti-)commutative theories are constructed by several deformations of usual (anti-)commutative SUSY theories. We will treat 2 types of the deformations given by SUSY $*$ product and non-SUSY $\star$ product. (These definitions will be given in the next section.) The SUSY $*$ product does not break the SUSY algebra, but SUSY $*$ products of chiral superfields are not chiral usually, then 1/2 SUSY is broken (see, for example, [10]). On the other hand, non-SUSY $\star$ products of chiral superfields are chiral superfields, but the SUSY algebra itself is broken as we saw in [9] and so on.

In this article, we will show that tuning of coupling constants regains full SUSY for not only SUSY $*$ deformation cases but also non-SUSY $\star$ deformation cases where the normal SUSY algebra is broken. In non(anti-)commutative theories, all lagrangians, algebras and transformation laws should be defined by using $*$ (or $\star$ etc.) products. Since the $*$ (or $\star$ etc.) product often breaks SUSY, it is non-trivial question to ask that one can define the SUSY algebra in non(anti-)commutative superspaces or there exist non(anti-)commutative full SUSY lagrangians. At first we will propose a general method to construct the full SUSY Wess-Zumino models in non(anti-)commutative superspaces. Using this procedure, we will understand that full SUSY recovers at specific values of the coupling constants. After that we will observe the quantum effect around the supersymmetry recovering points. The quantum corrections will be calculated and β-functions will be determined at one-loop order. In some phases, full SUSY is stable in the IR limit.

2. Conventions

We study the Wess-Zumino models on 4-dim euclidean superspace and use the convention of [62]. Non(anti-)deformation is given by several ways. For example, in [10], we can see the systematic explanation of the deformations. So we use the notation of [10].

The left covariant derivative is identical to the ordinary supersymmetric covariant derivative,

$$\overrightarrow{D}\Phi := D\Phi\,. \tag{2.1}$$

On the other hand, the right covariant derivative is defined through the following relation.

$$\Phi\overleftarrow{D} := (-1)^{p_D(p_\Phi+1)}\overrightarrow{D}\Phi\,, \tag{2.2}$$

where p_A is parity of A i.e. $p_A = 0$ for bosonic A and $p_B = 1$ for fermionic B. With this setting, for some superfields Φ and Ψ the SUSY $*$ product is defined by

$$\begin{aligned}\Phi * \Psi &:= \Phi\cdot\Psi + P^{\alpha\beta}\Phi\overleftarrow{D}_\alpha\overrightarrow{D}_\beta\Psi + \frac{1}{4}\det P\Phi\overleftarrow{D}^2\overrightarrow{D}^2\Psi\\ &= \Phi\cdot\Psi - P^{\alpha\beta}D_\alpha\Phi D_\beta\Psi - \frac{1}{4}\det PD^2\Phi D^2\Psi\,.\end{aligned} \tag{2.3}$$

This $*$ product is naively extended to the product of operators. For arbitrary operators (or superfields) O_1 and O_2,

$$\begin{aligned}O_1 * O_2 &= O_1\cdot O_2 + (-)^{p_D(p_{O1}+1)}P^{\alpha\beta}\,[D_\alpha, O_1\}[D_\beta, O_2\} -\\ &\quad - \frac{1}{4}\det P\epsilon^{\beta\alpha}\,[D_\alpha,[D_\beta,O_1\}\}\,\epsilon^{\delta\gamma}\,[D_\gamma,[D_\delta,O_2\}\}\,,\end{aligned} \tag{2.4}$$

where $[A, B\} := AB - (-)^{p_A p_B}BA$.

We introduce another typical non(anti-)commutative deformation with using not D but Q. The left SUSY generating operator action is identical to the ordinary action

$$\overrightarrow{Q}\Phi := Q\Phi\,. \tag{2.5}$$

On the other hand, the right action is defined by

$$\Phi\overleftarrow{Q} := (-1)^{p_Q(p_\Phi+1)}\overrightarrow{Q}\Phi\,. \tag{2.6}$$

With this setting, the non-SUSY $\star$ product is defined by [1]

$$\begin{aligned}\Phi \star \Psi &:= \Phi\cdot\Psi + P^{\alpha\beta}\Phi\overleftarrow{Q}_\alpha\overrightarrow{Q}_\beta\Psi + \frac{1}{4}\det P\Phi\overleftarrow{Q}^2\overrightarrow{Q}^2\Psi\\ &= \Phi\cdot\Psi - P^{\alpha\beta}Q_\alpha\Phi Q_\beta\Psi - \frac{1}{4}\det PQ^2\Phi Q^2\Psi\,.\end{aligned} \tag{2.7}$$

[1] In this article, the right actions are defined by (2.2) and (2.6), and these definitions are different from [9]. Therefore, the sign in the second equation in (2.7) is not same as the $\star$ product expression in [9]. But this difference is not important and does not affect following discussions.

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3. Full SUSY in non(anti-)commutative field theory

In this section, we propose a general procedure to build full SUSY lagrangians in non(anti-)commutative $\mathbb{R}^4$ from 1/2 SUSY lagrangians.

We will begin by considering the SUSY algebra determined by the SUSY $*$ product.

$$\{Q_\alpha, Q_\beta\}_* = \{\bar{Q}_{\dot\alpha}, \bar{Q}_{\dot\beta}\}_* = 0 \tag{3.1}$$

$$\{Q_\alpha, \bar{Q}_{\dot\beta}\}_* = 2\sigma^\mu_{\alpha\dot\beta} P_\mu = -2i\sigma^\mu_{\alpha\dot\beta}\partial_\mu \tag{3.2}$$

$$[P_\mu, Q_\alpha]_* = \left[P_\mu, \bar{Q}_{\dot\alpha}\right]_* = 0 \tag{3.3}$$

$$[M_{\mu\nu}, Q_\alpha]_* = -(i\sigma_{\mu\nu})_\alpha^\beta Q_\beta \tag{3.4}$$

$$\left[M_{\mu\nu}, \bar{Q}^{\dot\alpha}\right]_* = -(i\bar\sigma_{\mu\nu})_{\dot\alpha}^{\dot\beta} \bar{Q}^{\dot\beta}\,, \tag{3.5}$$

where $\{A, B\}_* := A * B + B * A$ and $[A, B]_* := A * B - B * A$. The definition of SUSY $*$ product (2.3) uses the covariant derivative D_α, and all generators of supersymmetric Poincare group commute with D_α, therefore the above SUSY algebra is trivial. We call the symmetry generated by the above algebra super-$*$ symmetry, i.e. a super-$*$ symmetric lagrangian L satisfies that

$$\int d^4x\ Q_\alpha * L = 0\,, \qquad \int d^4x\ \bar{Q}_{\dot\alpha} * L = 0\,. \tag{3.6}$$

Note that the super-$*$ symmetry is defined in the non(anti-)commutative superspace.

Let us construct full SUSY Wess-Zumino like models in the non(anti-)commutative Euclidean space that is deformed by the SUSY $*$ product. Let Φ be a chiral super field. Let $\int d^4\theta V^0(\Phi;*)\bar\theta^2$ be a 1/2 SUSY invariant lagrangian, where the $V^0(\Phi;*)$ is a some polynomial in Φ and its derivative $\partial_\mu\Phi$, $D_\alpha\Phi$ and so on. A general 1/2 SUSY lagrangian is given from the elements of the 1/2 SUSY ring discussed in [55]. In $V^0(\Phi;*)$, all products are $*$ products, but $V^0(\Phi;*)$ is possible to be expressed by usual products by using (2.3). Let $\tilde{V}^0(\Phi;\cdot)$ denote the usual product representation of $V^0(\Phi;*)$ i.e. $V^0(\Phi;*) = \tilde{V}^0(\Phi;\cdot)$. We can regard $\tilde{V}^0(\Phi;\cdot)$ as the sum of supersymmetric lagrangian $\tilde{V}^0_s(\Phi;\cdot)$ and 1/2 SUSY lagrangian $-\tilde{V}^0_{1/2}(\Phi;\cdot)$:

$$\tilde{V}^0(\Phi;\cdot) = \tilde{V}^0_s(\Phi;\cdot) - \tilde{V}^0_{1/2}(\Phi;\cdot)\,, \tag{3.7}$$

where $\tilde{V}^0_s(\Phi;\cdot)$ and $\tilde{V}^0_{1/2}(\Phi;\cdot)$ satisfy

$$\int d^4x d^4\theta\bar\theta^2 Q_\alpha \tilde{V}^0_s(\Phi;\cdot) = 0\,, \qquad \int d^4x d^4\theta\bar\theta^2 \bar{Q}_{\dot\alpha} \tilde{V}^0_s(\Phi;\cdot) = 0 \tag{3.8}$$

$$\int d^4x d^4\theta\bar\theta^2 Q_\alpha \tilde{V}^0_{1/2}(\Phi;\cdot) = 0\,, \qquad \int d^4x d^4\theta\bar\theta^2 \bar{Q}_{\dot\alpha} \tilde{V}^0_{1/2}(\Phi;\cdot) \neq 0\,. \tag{3.9}$$

Next step, we introduce a new lagrangian $\int d^4\theta\bar\theta^2 V^1(\Phi;*)$ by

$$\int d^4\theta \{V^1(\Phi;*)\}\bar\theta^2 := \int d^4\theta\{V^0(\Phi;*) + V^0_{1/2}(\Phi;*)\}\bar\theta^2\,, \tag{3.10}$$

where $V^0_{1/2}(\Phi;*)$ is $\tilde{V}^0_{1/2}(\Phi;\cdot)$ deformed by replacing all usual products with SUSY $*$ products. From (2.3),

$$\begin{aligned} V^0_{1/2}(\Phi;*) &= \tilde{V}^0_{1/2}(\Phi;\cdot) + (\text{higher order of } P^{\alpha\beta}) \\ &:= \tilde{V}^0_{1/2}(\Phi;\cdot) + \tilde{V}^1_{\rm def}(\Phi, P^{\alpha\beta};\cdot). \end{aligned} \tag{3.11}$$

In general, we can divide $\tilde{V}^1_{def}(\Phi, P^{\alpha\beta};\cdot)$ into a full SUSY part $\tilde{V}^1_s(\Phi, P^{\alpha\beta};\cdot)$ and a 1/2 SUSY part $\tilde{V}^1_{1/2}(\Phi, P^{\alpha\beta};\cdot)$:

$$\tilde{V}^1_{\rm def}(\Phi, P^{\alpha\beta};\cdot) = \tilde{V}^1_s(\Phi, P^{\alpha\beta};\cdot) - \tilde{V}^1_{1/2}(\Phi, P^{\alpha\beta};\cdot)\,, \tag{3.12}$$

where $\tilde{V}^1_s(\Phi, P^{\alpha\beta};\cdot)$ and $\tilde{V}^1_{1/2}(\Phi, P^{\alpha\beta};\cdot)$ satisfy

$$\int d^4x d^4\theta\bar{\theta}^2 Q_\alpha \tilde{V}^1_s(\Phi, P^{\alpha\beta};\cdot) = 0\,, \qquad \int d^4x d^4\theta\bar{\theta}^2 \bar{Q}_{\dot\alpha} \tilde{V}^1_s(\Phi, P^{\alpha\beta};\cdot) = 0 \tag{3.13}$$

$$\int d^4x d^4\theta\bar{\theta}^2 Q_\alpha \tilde{V}^1_{1/2}(\Phi, P^{\alpha\beta};\cdot) = 0\,, \qquad \int d^4x d^4\theta\bar{\theta}^2 \bar{Q}_{\dot\alpha} \tilde{V}^1_{1/2}(\Phi, P^{\alpha\beta};\cdot) \neq 0\,. \tag{3.14}$$

From (3.10), (3.11) and (3.12), the lagrangian is

$$\int d^4\theta\{V^1(\Phi;*)\}\bar{\theta}^2 = \int d^4\theta\bar{\theta}^2\{[\tilde{V}^0_s(\Phi;\cdot) + \tilde{V}^1_s(\Phi, P^{\alpha\beta};\cdot)] - \tilde{V}^1_{1/2}(\Phi, P^{\alpha\beta};\cdot)\}\,. \tag{3.15}$$

Note that $\tilde{V}^0_s(\Phi;\cdot) + \tilde{V}^1_s(\Phi, P^{\alpha\beta};\cdot)$ in the right hand side is full supersymmetric. We can duplicate the process from (3.10) to (3.15) to eliminate the 1/2 SUSY terms. The n-th process is as follows. The n-th lagrangian is

$$\int d^4\theta\{V^n(\Phi;*)\}\bar{\theta}^2 := \int d^4\theta\{V^{n-1}(\Phi;*) + V^{n-1}_{1/2}(\Phi;*)\}\bar{\theta}^2\,, \tag{3.16}$$

where $V^{n-1}_{1/2}(\Phi;*)$ is $\tilde{V}^{n-1}_{1/2}(\Phi;\cdot)$ deformed by replacing usual products by SUSY $*$ products. From (2.3),

$$\begin{aligned} V^{n-1}_{1/2}(\Phi;*) &= \tilde{V}^{n-1}_{1/2}(\Phi, P^{\alpha\beta};\cdot) + (\text{higher order of } P^{\alpha\beta}) \\ &:= \tilde{V}^{n-1}_{1/2}(\Phi;\cdot) + \tilde{V}^n_{\rm def}(\Phi, P^{\alpha\beta};\cdot)\,. \end{aligned} \tag{3.17}$$

We divide $\tilde{V}^n_{\rm def}(\Phi, P^{\alpha\beta};\cdot)$ into a full SUSY part $\tilde{V}^n_s(\Phi, P^{\alpha\beta};\cdot)$ and a 1/2 SUSY part $\tilde{V}^n_{1/2}(\Phi, P^{\alpha\beta};\cdot)$. Using this, the n-th lagrangian is given as

$$\begin{aligned} \int d^4\theta\{V^n(\Phi;*)\}\bar{\theta}^2 = \int d^4\theta\bar{\theta}^2\{[\tilde{V}^0_s(\Phi;\cdot) + \tilde{V}^1_s(\Phi, P^{\alpha\beta};\cdot) + \cdots + \tilde{V}^n_s(\Phi, P^{\alpha\beta};\cdot)] - \\ - \tilde{V}^n_{1/2}(\Phi, P^{\alpha\beta};\cdot)\}\,. \end{aligned} \tag{3.18}$$

The key point is these processes get over at finite rotation. Because the deformation of (3.16) makes $\tilde{V}^n_{1/2}(\Phi, P^{\alpha\beta};\cdot)$ be higher order terms of $P^{\alpha\beta}$. In proportion to the square root of the power of $P^{\alpha\beta}$, the number of D_α in $\tilde{V}^n_{1/2}(\Phi, P^{\alpha\beta};\cdot)$ increases. Since $D_\alpha D_\beta D_\gamma = 0$, there is a finite number N such that $\tilde{V}^N_{1/2}(\Phi, P^{\alpha\beta};\cdot) = 0$.

Then we get the full SUSY (super-$*$ symmetric) lagrangian

$$\int d^4\theta\{V^N(\Phi;*)\}\bar\theta^2 = \int d^4\theta\bar\theta^2\{V^0(\Phi;*) + [V^0_{1/2}(\Phi;*) + \cdots + V^{N-1}_{1/2}(\Phi;*)]\} \tag{3.19}$$
$$= \int d^4\theta\bar\theta^2\{\tilde V^0_s(\Phi;\cdot) + \tilde V^1_s(\Phi, P^{\alpha\beta};\cdot) + \cdots + \tilde V^N_s(\Phi, P^{\alpha\beta};\cdot)\}.$$

We will take examples to illustrate the above method to construct full SUSY lagrangians.

The first example is

$$S = \int d^4xd^2\theta d^2\bar\theta\bar\Phi * \Phi + \int d^4xd^2\bar\theta\frac{\bar m}{2}\bar\Phi * \bar\Phi + \int d^4xd^2\theta\{\frac{m}{2}\Phi * \Phi + g_{0*}\Phi * \Phi * \Phi\}. \tag{3.20}$$

The quadratic terms are not deformed by the SUSY $*$ product. Then the only $\Phi * \Phi * \Phi$ should be modified to get the full SUSY action. Let us follow the above instruction in the condition $\Phi * \Phi * \Phi = V^0(\Phi;*)$. Rewriting this as

$$\Phi * \Phi * \Phi = \Phi^3 - \frac{1}{4}\det P\Phi D^2\Phi D^2\Phi = \tilde V^0(\Phi;\cdot), \tag{3.21}$$

then $\tilde V^0_s(\Phi,\cdot) = \Phi^3$ and $\tilde V^0_{1/2}(\Phi,\cdot) = \frac{1}{4}\det P\Phi D^2\Phi D^2\Phi$. Thus,

$$V^0_{1/2}(\Phi,*) = \frac{1}{4}\det P\Phi * D^2_* * \Phi * D^2_* * \Phi, \tag{3.22}$$

where $D^2_* := D^\alpha * D_\alpha$. In the following, we often use that $D^2_* = D^2$, $D * \Phi = D\Phi$ etc. Using (3.22), a new lagrangian is introduced by

$$\int d^4\theta\bar\theta^2\{V^1(\Phi;*)\} = \int d^4\theta\bar\theta^2\{\Phi * \Phi * \Phi + \frac{1}{4}\det P\Phi * D^2\Phi * D^2\Phi\}$$
$$= \int d^4\theta\bar\theta^2\Phi^3. \tag{3.23}$$

This is the super-$*$ symmetric (full SUSY) lagrangian. This lagrangian is regarded as a special case of 1/2 SUSY lagrangian $\int d^4\theta\bar\theta^2\{g_{0*}\Phi * \Phi * \Phi + g_{1*}\Phi * D^2\Phi * D^2\Phi\}$, where g_{0*} and g_{1*} are coupling constants. From this point of view, we can observe that tuning coupling constants,

$$g_{1*}/g_{0*} \to \frac{1}{4}\det P,$$

realizes full SUSY.

The second example is the case of $\Phi^4_* = V^0(\Phi;*)$, where $\Phi^n_* = \underbrace{\Phi * \cdots * \Phi}_{n}$. From

$$\Phi^4_* = \Phi^4 - \frac{1}{4}\det P\Phi^2D^2\Phi D^2\Phi - \frac{1}{4}\det PD^2\Phi D^2\Phi^3 + \frac{1}{16}(\det P)^2(D^2\Phi)^4, \tag{3.24}$$

$\tilde V^0_s$ and $\tilde V^0_{1/2}$ are given by

$$\tilde V^0_s(\Phi,\cdot) = \Phi^4$$
$$\tilde V^0_{1/2}(\Phi,\cdot) = \frac{1}{4}\det P\Phi^2D^2\Phi D^2\Phi + \frac{1}{4}\det PD^2\Phi D^2\Phi^3 - \frac{1}{16}(\det P)^2(D^2\Phi)^4.$$

Therefore, the modified lagrangian is given by

$$\begin{aligned}\int d^4\theta\bar{\theta}^2 V^1(\Phi,*) &= \int d^4\theta\bar{\theta}^2\{V^0(\Phi,*) + V^0_{1/2}(\Phi,*)\}\\ &= \int d^4\theta\bar{\theta}^2\Big\{\Phi_*^4 + \Big[\frac{1}{4}\det P\Phi_*^2 D^2\Phi D^2\Phi + \frac{1}{4}\det P D^2\Phi D^2\Phi_*^3 - \\ &\qquad\qquad - \frac{1}{16}(\det P)^2(D^2\Phi)^4\Big]\Big\}\\ &= \int d^4\theta\bar{\theta}^2\left\{\Phi^4 - \frac{1}{8}(\det P)^2(D^2\Phi)^4\right\}\\ &= \int d^4\theta\bar{\theta}^2\Phi^4\,. \end{aligned} \tag{3.25}$$

This is what we want. Again, we can regard this lagrangian is given by tuning of coupling constants of the 1/2 SUSY lagrangian.

This method is extended easily to the case of non-SUSY $\star$ deformation. However, the SUSY algebra for non-SUSY $\star$ product has to be modified, because the $\star$ definition (2.7) uses Q_α and it does not commute with $\bar{Q}_{\dot{\alpha}}$. So we replace $\bar{Q}_{\dot{\alpha}}$ by $\bar{\mathcal{Q}}_{\dot{\alpha}}$ defined by[2]

$$\bar{\mathcal{Q}}_{\dot{\alpha}} := \bar{Q}_{\dot{\alpha}} + P^{\alpha\beta}\{Q_\alpha, \bar{Q}_{\dot{\alpha}}\}Q_\beta = \bar{Q}_{\dot{\alpha}} - 2P^{\alpha\beta}(i\sigma^\mu_{\alpha\dot{\alpha}}\partial_\mu)Q_\beta\,. \tag{3.26}$$

From this definition, $\bar{\mathcal{Q}}_{\dot{\alpha}}$ satisfies

$$\bar{\mathcal{Q}}_{\dot{\alpha}} \star \phi = \bar{Q}_{\dot{\alpha}}\phi\,, \tag{3.27}$$

for arbitrary field $\phi = \phi(x,\theta,\bar{\theta})$. Using $\bar{\mathcal{Q}}_{\dot{\alpha}}$, the super-$\star$ symmetry algebra is defined by

$$\{Q_\alpha, Q_\beta\}_\star = \{\bar{\mathcal{Q}}_{\dot{\alpha}}, \bar{\mathcal{Q}}_{\dot{\beta}}\}_\star = 0 \tag{3.28}$$

$$\{Q_\alpha, \bar{\mathcal{Q}}_{\dot{\beta}}\}_\star = 2\sigma^\mu_{\alpha\dot{\beta}}P_\mu = -2i\sigma^\mu_{\alpha\dot{\beta}}\partial_\mu \tag{3.29}$$

$$[P_\mu, Q_\alpha]_\star = \left[P_\mu, \bar{\mathcal{Q}}_{\dot{\alpha}}\right]_\star = 0 \tag{3.30}$$

$$[M_{\mu\nu}, Q_\alpha]_\star = -(i\sigma_{\mu\nu})_\alpha^\beta Q_\beta \tag{3.31}$$

$$\left[M_{\mu\nu}, \bar{\mathcal{Q}}^{\dot{\alpha}}\right]_\star = -(i\bar{\sigma}_{\mu\nu})^{\dot{\beta}}_{\dot{\alpha}}\bar{\mathcal{Q}}^{\dot{\beta}}\,. \tag{3.32}$$

Here $\{A,B\}_\star := A\star B + B\star A$ and $[A,B]_\star := A\star B - B\star A$. Note that $\bar{\mathcal{Q}}_{\dot{\alpha}}$ is not derivative because it includes second derivative and does not satisfy the Leibniz rule. So, we can not make a group from the above super-$\star$ symmetry algebra. However, it is possible to make full super-$\star$ symmetric field theories, where the super-$\star$ symmetry is defined by invariance under

$$\begin{aligned}\Phi &\to \Phi + \zeta^\alpha Q_\alpha \star \Phi + \bar{\zeta}_{\dot{\alpha}}\bar{\mathcal{Q}}^{\dot{\alpha}} \star \Phi = \Phi + \zeta^\alpha Q_\alpha\Phi + \bar{\zeta}_{\dot{\alpha}}\bar{Q}^{\dot{\alpha}}\Phi\,,\\ \bar{\Phi} &\to \bar{\Phi} + \zeta^\alpha Q_\alpha \star \bar{\Phi} + \bar{\zeta}_{\dot{\alpha}}\bar{\mathcal{Q}}^{\dot{\alpha}} \star \bar{\Phi} = \bar{\Phi} + \zeta^\alpha Q_\alpha\bar{\Phi} + \bar{\zeta}_{\dot{\alpha}}\bar{Q}^{\dot{\alpha}}\bar{\Phi}\,,\end{aligned}$$

where ζ and $\bar{\zeta}$ are fermionic spinor parameters. This super-$\star$ symmetry is defined as a symmetry of non(anti-)commutative field theory. As mentioned above, this transformation does not mean the super Poincare group transformation of the superspace, because $\bar{\mathcal{Q}}_{\dot{\alpha}}$ is

[2]This $\bar{\mathcal{Q}}_{\dot{\alpha}}$ first appeared in [9].

not derivative and it does not make a group action, in the sense of non(anti-)commutative theory. Therefore, this symmetry is not a superspace symmetry but a symmetry defined by an infinitesimal field transformation. However, arbitrary super-$\star$ symmetric lagrangians satisfy

$$\int d^4x Q_\alpha \star L = 0\,, \qquad \int d^4x \bar{Q}_{\dot\alpha} \star L = 0\,. \tag{3.33}$$

We can construct super-$\star$ symmetric lagrangians by the same way of above super-$*$ symmetric lagrangian construction.

There are some comments. The method in this section is formally extended to other 1/2 SUSY theories in non(anti-)commutative superspaces. We expect, for example, 1/2 SUSY gauge theories are deformed to full SUSY theories by this method. However, there may be some problems on eliminating SUSY breaking terms. There are no assurance that this process stop at finite rotation, for some kind of theories like non-abelian gauge theories. Also, it is not clear that the (deformed) gauge invariance is maintained in this procedure. Therefore, more detailed analyses are needed.

The super-$*(\star)$ symmetric action in non(anti-)commutative superspace is equivalent to the normal SUSY action given by $V^0(\Phi; *(\star))$ replacing $*(\star)$ with the usual multiplication, in above two examples. It may be that there are equivalent non(anti-)commutative theories to arbitrary SUSY theories in usual space.

Considering Wess-Zumino models, our method produce Φ^n interaction term in commutative theory when we start from $V^0(\Phi; *) = \Phi_0^n$. But it remains an unsettled question whether there exists such simple correspondence for any models.

In this article, we treat the $*$ and $\star$ deformed theories. There are many kinds of non(anti-)commutative deformations, and it is known that some deformations do not break any SUSY (see, for example, [42]). In any case, when deformed products are expressed by a polynomial with finite terms, the above method is valid to construct full SUSY non(anti-)commutative lagrangians.

4. 1-loop Calculations

In the previous section, we gave the prescription to obtain full SUSY actions in non(anti-)commutative superspaces. In this section, we will investigate quantum effects [11]–[22], concentrating on the Φ_*^3 model for simplicity.

From the results of the previous section, we know that

$$\begin{aligned} S_. = & \int d^4x d^2\theta d^2\bar\theta \bar\Phi\Phi + \int d^4x d^2\theta d^2\bar\theta\theta^2 \frac{\bar m}{2}\bar\Phi^2 + \\ & + \int d^4x d^2\theta d^2\bar\theta\bar\theta^2 \left\{ \frac{m}{2}\Phi^2 + g_0\Phi^3 + g_1 \Phi D^2\Phi D^2\Phi \right\}, \end{aligned} \tag{4.1}$$

and

$$\begin{aligned} S_* = & \int d^4x d^2\theta d^2\bar\theta \bar\Phi * \Phi + \int d^4x d^2\theta d^2\bar\theta\theta^2 \frac{\bar m}{2}\bar\Phi_*^2 + \\ & + \int d^4x d^2\theta d^2\bar\theta\bar\theta^2 \left\{ \frac{m}{2}\Phi_*^2 + g_{0*}\Phi_*^3 + g_{1*}\Phi * D_*^2 * \Phi * D_*^2 * \Phi \right\}, \end{aligned} \tag{4.2}$$

are equivalent, if

$$g_0 = g_{0*}\,, \qquad g_1 = g_{1*} - \frac{1}{4}\det P\ g_{0*}\,. \tag{4.3}$$

We will calculate 1-loop graphs and β-functions based on (4.1),[3] and interpret the results in terms of (4.2).

Before calculating 1-loop corrections, it is convenient to integrate out $\bar{\Phi}$ in (4.1) [12, 61].

$$S = \int d^4x d^2\theta d^2\bar{\theta}\bar{\theta}^2 \left\{ \frac{1}{2}\Phi\left(m - \frac{\Box}{\bar{m}}\right)\Phi + g_0\Phi^3 + g_1\Phi D^2\Phi D^2\Phi \right\}. \tag{4.4}$$

Also we have to add a term proportional to $\Phi D^2\Phi$ to renormalize a 1-loop divergent graph (See figure 1) [11, 12]. So we take the following action as the starting point:

$$S = \int d^4x d^2\theta d^2\bar{\theta}\bar{\theta}^2 \left\{ \frac{1}{2}\Phi_b\left(\frac{1}{4\lambda_b}D^2 + m_b - \frac{\Box}{\bar{m}_b}\right)\Phi_b + g_{0b}\Phi_b^3 + g_{1b}\Phi_b D^2\Phi_b D^2\Phi_b \right\}, \tag{4.5}$$

where the subscription b denotes Φ_b etc. are bare quantities. In [16]–[19], the renormalizability of (4.5) was proved.[4]

To renormalize 1-loop corrections, we define the following renormalized quantities:

$$\begin{aligned} \Phi_b &= \sqrt{Z_\Phi}(\Phi_r + \delta\Phi)\,, & m_b &= m_r + \delta m\,, & \bar{m}_b &= Z_{\bar{m}} Z_\Phi \bar{m}_r\,, \\ \lambda_b &= Z_\lambda Z_\Phi \lambda_r\,, & g_{0b} &= Z_0 \sqrt{Z_\Phi}^{-3} g_{0r}\,, & g_{1b} &= Z_1 \sqrt{Z_\Phi}^{-3} g_{1r}\,, \end{aligned} \tag{4.6}$$

where

$$\begin{aligned} Z_\Phi &= 1 + Z_\Phi^{(1)} + \cdots\,, & \delta\Phi &= 0 + \delta\Phi^{(1)} + \cdots\,, \\ \delta m &= 0 + \delta m^{(1)} + \cdots\,, & Z_{\bar{m}} &= 1 + Z_{\bar{m}}^{(1)} + \cdots\,, \\ Z_\lambda &= 1 + Z_\lambda^{(1)} + \cdots\,, & Z_0 &= 1 + Z_0^{(1)} + \cdots\,, & Z_1 &= 1 + Z_1^{(1)} + \cdots\,. \end{aligned} \tag{4.7}$$

1-loop calculations tell us

$$\begin{aligned} \delta m &= 0\,, & Z_{\bar{m}}^{(1)} &= 0\,, \\ Z_0^{(1)} &= 0\,, & Z_1^{(1)} &= 0\,, \end{aligned} \tag{4.8}$$

and

$$\frac{1}{2}Z_\Phi^{(1)} m_r + 3g_{0r}\delta\Phi^{(1)} = 0\,, \tag{4.9}$$

so we find Z_λ and $\delta\Phi$ are independent. They are determined by the renormalization for 1-loop divergent graphs, figure 1 and figure 2.

[3] In [14], β-functions based on slightly different action were calculated at 2-loop level. Also, in [20], β-functions for the non(anti-)commutative gauge theory were calculated at 1-loop level.

[4] In [16, 17, 18], the proof is based on the global $U(1)$ symmetries and graphical arguments. On the other hand, the proof in [19] is based on observation that one can adjust the dimensions of the fermionic coordinates to make the non(anti-)commutative parameter $P^{\alpha\beta}$ dimensionless. Both are safely applied to the case of (4.5), a special case of $\bar{g}_0 = \bar{g}_1 = 0$.

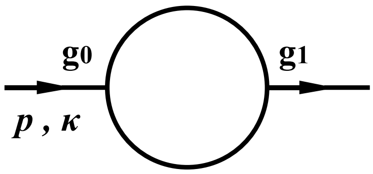

Figure 1: 1-loop divergent graph contributing to $\frac{1}{4\lambda}\Phi D^2\Phi$.

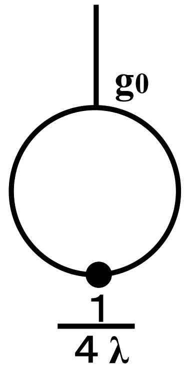

Figure 2: 1-loop divergent tadpole graph canceled by $\delta\Phi$.

It is worthwhile to comment on the role of $\frac{1}{4\lambda}\Phi D^2\Phi$ in (4.5). Firstly, as we mentioned above, this is necessary to renormalize the 1-loop divergent graph, figure 1. Secondly, this term changes the propagator:

$$\begin{aligned} \frac{\bar m}{m\bar m + p^2} \Rightarrow & \frac{\bar m}{m\bar m + p^2 + \frac{\bar m}{4\lambda}\kappa^2}\,, \\ = & \frac{\bar m}{m\bar m + p^2} - \frac{\bar m}{m\bar m + p^2}\frac{\kappa^2}{4\lambda}\frac{\bar m}{m\bar m + p^2}\,, \end{aligned} \tag{4.10}$$

where κ is the conjugate momentum of the fermionic coordinate θ. This change yields the divergent tadpole graph, figure 2, at the 1-loop level. This non-vanishing tadpole causes the field shift $\delta\Phi$, and the wave function renormalization through the relation (4.9). It is shown that the above field shift does not lift the vacuum energy, then it does not break the (1/2) SUSY [11, 12].

We adopt the dimensional regularization, that is, we replace the dimension 4 by $n \in \mathbb{C}$, and the minimal subtraction renormalization. So we introduce a scale parameter μ whose mass dimension is 1 and redefine the renormalized quantities by

$$m_b = \mu \tilde m_r\,, \tag{4.11}$$
$$\bar m_b = Z_\Phi \mu \tilde{\bar m}_r\,, \tag{4.12}$$
$$\lambda_b = Z_\lambda Z_\Phi \lambda_r\,, \tag{4.13}$$
$$g_{0b} = Z_\Phi^{-\frac{3}{2}} \mu^{\frac{4-n}{2}} g_{0r}\,, \tag{4.14}$$
$$g_{1b} = Z_\Phi^{-\frac{3}{2}} \mu^{\frac{-n}{2}} \tilde g_{1r}\,. \tag{4.15}$$

Here, $\tilde m_r$, $\tilde{\bar m}_r$, λ_r, g_{0r} and $\tilde g_{1r}$ are dimensionless.

Let us determine Z_λ. The contribution of Fig.1 is

$$\Gamma^{(2)}_{D^2} = -\frac{9 g_{0r}\tilde g_{1r}\tilde{\bar m}_r^2}{\pi^2}\kappa^2\left(\frac{1}{\epsilon} + \cdots\right), \tag{4.16}$$

where $\epsilon = \frac{4-n}{2}$. The divergent part of (4.16) is canceled by the counter term $\frac{Z_\lambda^{(1)}}{4\lambda_r}\Phi_r D^2 \Phi_r$, so

$$Z_\lambda^{(1)} = \frac{1}{\epsilon}\left(-\frac{36 g_{0r}\tilde{g}_{1r}\tilde{\bar{m}}_r^2 \lambda_r}{\pi^2}\right). \tag{4.17}$$

Now we turn to $\delta\Phi$ and Z_Φ. The contribution of figure 2 is

$$\Gamma^{(1)} = \frac{3 g_{0r}\mu^2 \tilde{\tilde{m}}_r^2}{16\pi^2 \lambda_r}\left(\frac{1}{\epsilon} + \cdots\right). \tag{4.18}$$

The divergent part of (4.18) is canceled by the counter term $\mu \tilde{m}_r \delta\Phi^{(1)}$. From (4.9), we obtain

$$Z_\phi^{(1)} = \frac{1}{\epsilon}\left(\frac{9 g_{0r}^2 \tilde{\tilde{m}}_r^2}{8\pi^2 \tilde{m}_r^2 \lambda_r}\right). \tag{4.19}$$

Using (4.17) and (4.19), we calculate the β-functions defined by

$$\begin{aligned}
\beta_0 &= \frac{\partial g_{0r}}{\partial \log \mu}\,, \qquad \beta_{\tilde{1}} = \frac{\partial \tilde{g}_{1r}}{\partial \log \mu}\,, \qquad \beta_{\lambda^{-1}} = \frac{\partial \lambda_r^{-1}}{\partial \log \mu}\,, \\
\beta_{\tilde{m}} &= \frac{\partial \tilde{m}_r}{\partial \log \mu}\,, \qquad \beta_{\tilde{\tilde{m}}} = \frac{\partial \tilde{\tilde{m}}_r}{\partial \log \mu}\,.
\end{aligned} \tag{4.20}$$

The results are

$$\begin{aligned}
\beta_0 &= -\frac{27 g_{0r}^3 \lambda_r^{-1} \tilde{\tilde{m}}_r^2}{8\pi^2 \tilde{m}_r^2}\,, \qquad \beta_{\tilde{1}} = 2\tilde{g}_{1r} - \frac{27 g_{0r}^2 \tilde{g}_{1r} \lambda_r^{-1} \tilde{\tilde{m}}_r^2}{8\pi^2 \tilde{m}^2}\,, \\
\beta_{\lambda^{-1}} &= -\frac{9 g_{0r}^2 \lambda_r^{-2} \tilde{\tilde{m}}_r^2}{4\pi^2 \tilde{m}_r^2} + \frac{72 g_{0r}\tilde{g}_{1r}\tilde{\tilde{m}}_r^2}{\pi^2}\,, \\
\beta_{\tilde{m}} &= -\tilde{m}_r\,, \qquad \beta_{\tilde{\tilde{m}}} = -\tilde{\tilde{m}}_r - \frac{9 g_{0r}^2 \lambda_r^{-1} \tilde{\tilde{m}}_r^3}{4\pi^2 \tilde{m}_r^2}\,.
\end{aligned} \tag{4.21}$$

5. Discussions

In this section, we argue the RG behavior of Φ_*^3 model, particularly, around points realizing full SUSY. Notice that full SUSY is recovered when $g_1 = 0$ and $\lambda^{-1} = 0$. Naively, since g_1 has mass dimension -2, we expect full SUSY is repaired in the IR limit. However, as we will see in the following, the RG behavior of λ^{-1} modifies this speculation.

It makes things clear to classify situations according to the sign of λ^{-1}. From (4.21),

(i) $\lambda^{-1} > 0$ (figures 3–5). For g_0, $g_0 \to 0$ in the UV limit. For λ^{-1},

- $\lambda^{-1} \to 0$ in IR, if $\{g_0 > 0,\ g_1 > \frac{\lambda^{-2}}{32m^2} g_0\}$ or $\{g_0 < 0\,,\ g_1 < \frac{\lambda^{-2}}{32m^2} g_0\}$,
- $\lambda^{-1} \to 0$ in UV, if $\{g_0 > 0\,,\ g_1 < \frac{\lambda^{-2}}{32m^2} g_0\}$ or $\{g_0 < 0\,,\ g_1 > \frac{\lambda^{-2}}{32m^2} g_0\}$.

(ii) $\lambda^{-1} = 0$ (figures 6–8). $\beta_0 = 0$. For λ^{-1},

- $\beta_{\lambda^{-1}} > 0$, if $\{g_0 > 0\,,\ g_1 > 0\}$ or $\{g_0 < 0\,,\ g_1 < 0\}$,

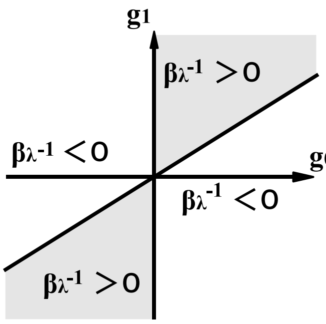

Figure 3: $\beta_{\lambda^{-1}} > 0$ in the grayed region. The oblique line represents $g_1 = \frac{\lambda^{-2}}{32m^2} g_0$.

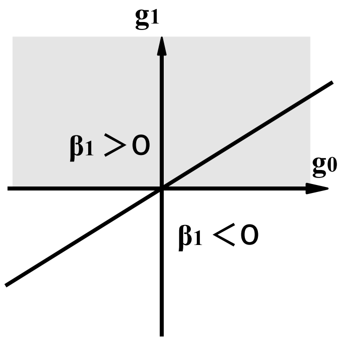

Figure 4: $\beta_{\tilde{1}} > 0$ in the grayed region. The oblique line represents $g_1 = \frac{\lambda^{-2}}{32m^2} g_0$.

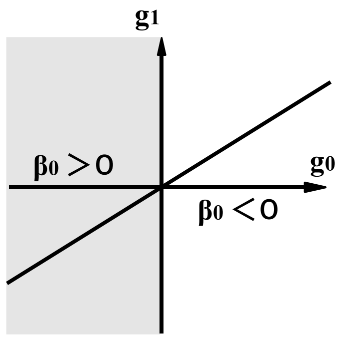

Figure 5: $\beta_0 > 0$ in the grayed region. The oblique line represents $g_1 = \frac{\lambda^{-2}}{32m^2} g_0$.

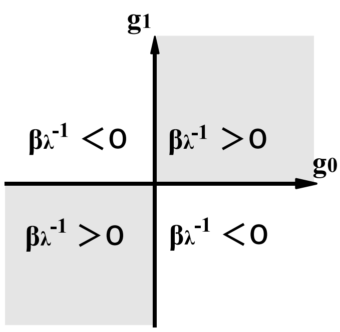

Figure 6: $\beta_{\lambda^{-1}} > 0$ in the grayed region.

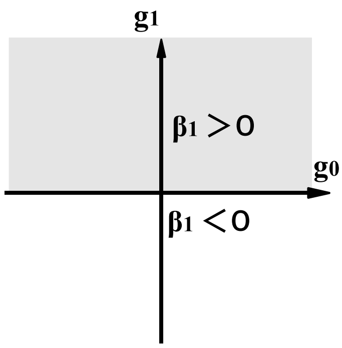

Figure 7: $\beta_{\tilde{1}} > 0$ in the grayed region.

- $\beta_{\lambda^{-1}} < 0$, if $\{g_0 > 0\,,\ g_1 < 0\}$ or $\{g_0 < 0\,,\ g_1 > 0\}$,
- $\beta_{\lambda^{-1}} = 0$, if $\{g_0 = 0\}$ or $\{g_1 = 0\}$.

(iii) $\lambda^{-1} < 0$ (figures 9–11). For g_0, $g_0 \to 0$ in the IR limit. For λ^{-1},

- $\lambda^{-1} \to 0$ in UV, if $\{g_0 > 0\,,\ g_1 > \frac{\lambda^{-2}}{32m^2} g_0\}$ or $\{g_0 < 0\,,\ g_1 < \frac{\lambda^{-2}}{32m^2} g_0\}$,
- $\lambda^{-1} \to 0$ in IR, if $\{g_0 > 0\,,\ g_1 < \frac{\lambda^{-2}}{32m^2} g_0\}$ or $\{g_0 < 0\,,\ g_1 > \frac{\lambda^{-2}}{32m^2} g_0\}$.

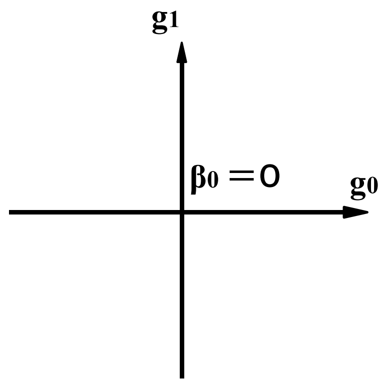

Figure 8: $\beta_0 = 0$ in the whole (perturbative) region.

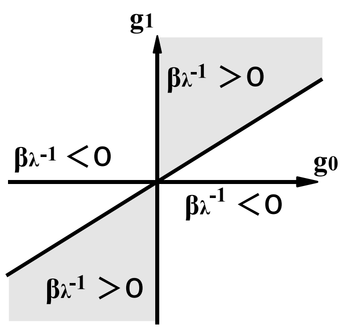

Figure 9: $\beta_{\lambda^{-1}} > 0$ in the grayed region. The oblique line represents $g_1 = \frac{\lambda^{-2}}{32m^2} g_0$.

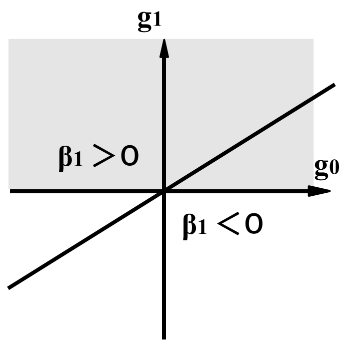

Figure 10: $\beta_{\tilde{1}} > 0$ in the grayed region. The oblique line represents $g_1 = \frac{\lambda^{-2}}{32m^2} g_0$.

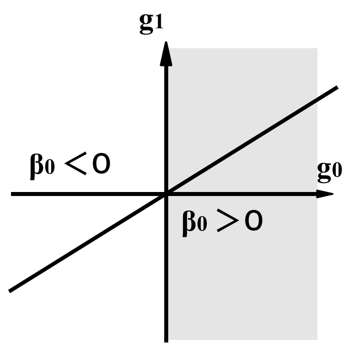

Figure 11: $\beta_0 > 0$ in the grayed region. The oblique line represents $g_1 = \frac{\lambda^{-2}}{32m^2} g_0$.

From the above results, full SUSY is realized in the IR limit if the RG flow starts from

$$\left\{ \lambda^{-1} > 0 \,,\ g_0 > 0 \,,\ g_1 > \frac{\lambda^{-2}}{32m^2} g_0 \right\}, \tag{5.1}$$

or

$$\left\{ \lambda^{-1} > 0 \,,\ g_0 < 0 \,,\ g_1 < \frac{\lambda^{-2}}{32m^2} g_0 \right\}, \tag{5.2}$$

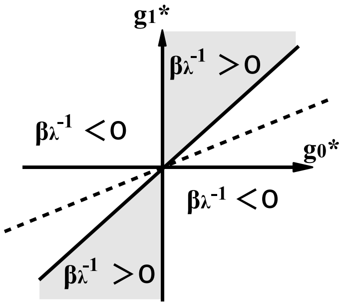

Figure 12: $\beta_{\lambda^{-1}} > 0$ in the grayed region. The dash oblique line represents $g_{1*} = \frac{1}{4}\det P g_{0*}$ and the solid oblique one does $g_{1*} = (\frac{1}{4}\det P + \frac{\lambda_*^{-2}}{32m^2})g_{0*}$.

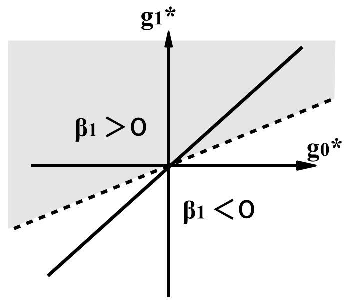

Figure 13: $\beta_{\tilde{1}} > 0$ in the grayed region. The dash oblique line represents $g_{1*} = \frac{1}{4}\det P g_{0*}$ and the solid oblique one does $g_{1*} = (\frac{1}{4}\det P + \frac{\lambda_*^{-2}}{32m^2})g_{0*}$.

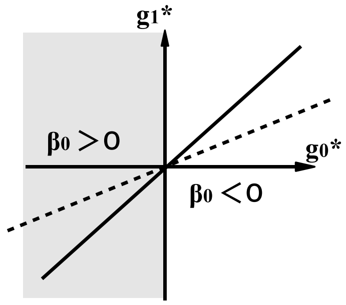

Figure 14: $\beta_0 > 0$ in the grayed region. The dash oblique line represents $g_{1*} = \frac{1}{4}\det P g_{0*}$ and the solid oblique one does $g_{1*} = (\frac{1}{4}\det P + \frac{\lambda_*^{-2}}{32m^2})g_{0*}$.

or

$$\left\{\lambda^{-1} < 0\,,\ g_0 > 0\,,\ g_1 < \frac{\lambda^{-2}}{32m^2}g_0\right\}, \tag{5.3}$$

or

$$\left\{\lambda^{-1} < 0\,,\ g_0 < 0\,,\ g_1 > \frac{\lambda^{-2}}{32m^2}g_0\right\}. \tag{5.4}$$

By using the relations (4.3) and $\lambda_* = \lambda$, the above argument can be rewritten in terms of the non(anti-)commutative field theory and the super-$*$ symmetry.

(i) $\lambda_*^{-1} > 0$ (figures 12–14). For g_{0*}, $g_{0*} \to 0$ in the UV limit. For λ_*^{-1},

- $\lambda_*^{-1} \to 0$ in IR,if $\{g_{0*} > 0\,,\ g_{1*} > (\frac{1}{4}\det P + \frac{\lambda_*^{-2}}{32m^2})g_{0*}\}$ or $\{g_{0*} < 0\,,\ g_{1*} < (\frac{1}{4}\det P + \frac{\lambda_*^{-2}}{32m^2})g_{0*}\}$,
- $\lambda_*^{-1} \to 0$ in UV, if $\{g_{0*} > 0,\ g_{1*} < (\frac{1}{4}\det P + \frac{\lambda^{-2}}{32m^2})g_{0*}\}$ or $\{g_{0*} < 0\,,\ g_{1*} > (\frac{1}{4}\det P + \frac{\lambda_*^{-2}}{32m^2})g_{0*}\}$.

(ii) $\lambda^{-1} = 0$ (figures 15–17). $\beta_0 = 0$. For λ^{-1},

- $\beta_{\lambda^{-1}} > 0$, if $\{g_{0*} > 0\,,\ g_{1*} > \frac{1}{4}\det P g_{0*}\}$ or $\{g_{0*} < 0\,,\ g_{1*} < \frac{1}{4}\det P g_{0*}\}$,
- $\beta_{\lambda^{-1}} < 0$, if $\{g_{0*} > 0,\ g_{1*} < \frac{1}{4}\det P g_{0*}\}$ or $\{g_{0*} < 0\,,\ g_{1*} > \frac{1}{4}\det P g_{0*}\}$,

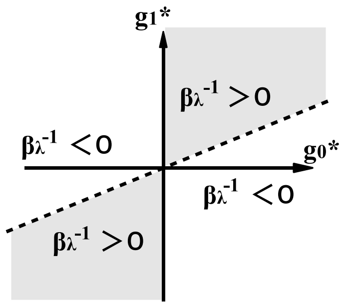

Figure 15: $\beta_{\lambda^{-1}} > 0$ in the grayed region. The dash oblique line represents $g_{1*} = \frac{1}{4}\det P g_{0*}$.

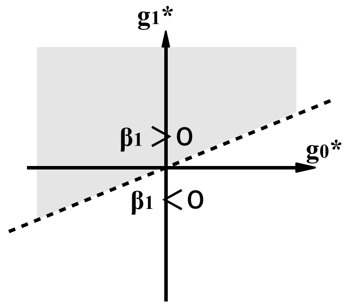

Figure 16: $\beta_{\tilde{1}} > 0$ in the grayed region. The dash oblique line represents $g_{1*} = \frac{1}{4}\det P g_{0*}$.

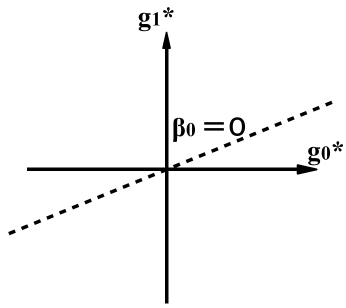

Figure 17: $\beta_0 = 0$ in the whole (perturbative) region. The dash oblique line represents $g_{1*} = \frac{1}{4}\det P g_{0*}$.

- $\beta_{\lambda^{-1}} = 0$, if $g_{0*} = 0$ or $g_{1*} = \frac{1}{4}\det P g_{0*}$.

(iii) $\lambda_*^{-1} < 0$ (figures 18–20). For g_{0*}, $g_{0*} \to 0$ in the IR limit. For λ_*^{-1},

- $\lambda_*^{-1} \to 0$ in UV, if $\{g_{0*} > 0,\ g_{1*} > (\frac{1}{4}\det P + \frac{\lambda_*^{-2}}{32m^2})g_{0*}\}$ or $\{g_{0*} < 0,\ g_{1*} < (\frac{1}{4}\det P + \frac{\lambda_*^{-2}}{32m^2})g_{0*}\}$,
- $\lambda_*^{-1} \to 0$ in IR, if $\{g_{0*} > 0,\ g_{1*} < (\frac{1}{4}\det P + \frac{\lambda_*^{-2}}{32m^2})g_{0*}\}$ or $\{g_{0*} < 0\,,\ g_{1*} > (\frac{1}{4}\det P + \frac{\lambda_*^{-2}}{32m^2})g_{0*}\}$.

From the above results, the super-$*$ symmetry is realized in the IR limit if the RG flow starts from

$$\left\{\lambda_*^{-1} > 0\,,\ g_{0*} > 0\,,\ g_{1*} > \left(\frac{1}{4}\det P + \frac{\lambda_*^{-2}}{32m^2}\right) g_{0*}\right\}, \tag{5.5}$$

or

$$\left\{\lambda_*^{-1} > 0\,,\ g_{0*} < 0\,,\ g_{1*} < \left(\frac{1}{4}\det P + \frac{\lambda_*^{-2}}{32m^2}\right) g_{0*}\right\}, \tag{5.6}$$

or

$$\left\{\lambda_*^{-1} < 0\,,\ g_{0*} > 0\,,\ g_{1*} < \left(\frac{1}{4}\det P + \frac{\lambda_*^{-2}}{32m^2}\right) g_{0*}\right\}, \tag{5.7}$$

or

$$\left\{\lambda_*^{-1} < 0\,,\ g_{0*} < 0\,,\ g_{1*} > \left(\frac{1}{4}\det P + \frac{\lambda_*^{-2}}{32m^2}\right) g_{0*}\right\}. \tag{5.8}$$

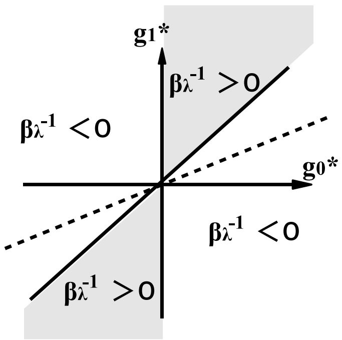

Figure 18: $\beta_{\lambda^{-1}} > 0$ in the grayed region. The dash oblique line represents $g_{1*} = \frac{1}{4} \det P g_{0*}$ and the solid oblique one does $g_{1*} = (\frac{1}{4} \det P + \frac{\lambda_*^{-2}}{32m^2}) g_{0*}$.

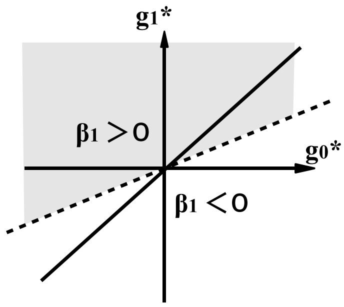

Figure 19: $\beta_{\bar{1}} > 0$ in the grayed region. The dash oblique line represents $g_{1*} = \frac{1}{4} \det P g_{0*}$ and the solid oblique one does $g_{1*} = (\frac{1}{4} \det P + \frac{\lambda_*^{-2}}{32m^2}) g_{0*}$.

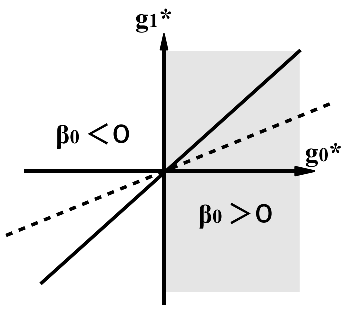

Figure 20: $\beta_0 > 0$ in the grayed region. The dash oblique line represents $g_{1*} = \frac{1}{4} \det P g_{0*}$ and the solid oblique one does $g_{1*} = (\frac{1}{4} \det P + \frac{\lambda_*^{-2}}{32m^2}) g_{0*}$.

6. Conclusions

In this article we gave the general prescription to construct full SUSY lagrangians in non(anti-)commutative superspaces, which is relevant for both the SUSY $*$-deformed superspace and the non-SUSY $\star$-deformed superspace.

We investigated quantum effects at the 1-loop level for the deformed Φ_*^3 Wess-Zumino model with the $\Phi D^2 \Phi$ proportional term. We found that this term yields a divergent tadpole graph and the renormalization of this causes the wave function renormalization. The wave function renormalization gave the non-trivial β- functions.

From the obtained β-functions, we found the conditions on the parameters (g_0, g_1, λ^{-1}) to recover full SUSY in the IR limit. We also rewrote the conditions into ones on $(g_{o*}, g_{1*},$ $\lambda_*^{-1})$, the parameters of the non(anti-)commutative field theories.

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